ISSN: 0975-8585

Research Journal of Pharmaceutical, Biological and Chemical Sciences

Mathematical Models for Financial Time Series Problems - A Review.

Seethalakshmi R[1]*, Vijayabanu C[2], Saavithri V[3], and Kannan K[4].

[1]Assistant Professor, Department of Mathematics, School of Humanities & Sciences, SASTRA University, Thanjavur.
[2]Associate Professor, School of Management, SASTRA University, Thanjavu.
[3]Professor, Department of Mathematics , Nehru Memorial College, Trichirappalli
[4]Professor, Department of Mathematics, School of Humanities & Sciences, SASTRA University, Thanjavur.

ABSTRACT

The stock Market volatility and its related prediction of risk factors are always challenging and bestow with research problems to Financial Mathematics research community. Models are developed according to the scenario prevailing in the market and the markets keep changing. Intelligent tools help to the extent of designing computational models to address certain issues to solve the problems from time to time. This study aims at developing mathematical models (Parameter driven and observation driven models) for a few financial time series problems to forecast volatility of stock markets. In this study, various mathematical models have been proposed for estimating financial volatility. In this research work, we confine our attention to discuss (A) two observation models (Stock well transform & HMM) for Volatility estimation and finding the risk-return series and (B) three parameter driven models namely (i) Two mixed distributions to examine their suitability for obtaining parameters fitting the data. (ii) An intelligent technique - Support Vector Machine (SVM) with PCA-algorithm, Volatility of stock market can very well predicted by the above mentioned select mathematical models which is a holistic and in depth understanding of usage of mathematical models in predicting stock market volatility / risk-return.

Keywords: Volatility, Hidden Markov model, Mixture distribution, Support Vector Machine, Stock well transform.

***Corresponding author**

ISSN: 0975-8585

INTRODUCTION

Volatility estimation in stock market problems is still a challenging area of research in Financial Mathematics (also called Time series problems). A wide variety of statistical models has been proposed by research community to address this issue. However, this area continues to pose much more challenges in the stock market scenario. Hence, there is a need for constructing computational models that can predict and estimate volatility.

The linearity in time series models as explained in earlier literature [1] are insufficient to represent time series problems related data in the closed form. So, there is a need to introduce a new class of models to deal with financial time series with the above features. **"Observation driven"** and **"parameter driven models",** may be two broad modules for financial time series modelling. The volatility (conditional variance) in Observation driven model is supposed to be a function of the past observations, which results in heteroscedacity in the model. Observation models include – Versions of ARIMA and Markov and Hidden Markov Models. Parameter driven models include - Generalized Linear ARMA, Various Versions of GARCH and other distribution based models. Intelligent techniques such as ANN (for classification and obtaining local optima), Probabilistic Neural Network, Fuzzy logic and Genetic algorithm (for obtaining global optima for historical volatility) can also be used for parametric / non-parametric optimization problems in Stock market forecasting problems.

Present Scenario

The present day stock market level is becoming unpredictable often due to frequent changes in price movements, Whipsaws etc. Due to its very large size, most of the times we need data reduction (or dimensionality reduction) techniques for obtaining good features for classification, prediction and estimation. For general time series problems, a number of data reduction techniques have been surveyed and a few techniques employable in the stock market scenario, their characteristics, merits and limitations are studied.

STATEMENT OF PROBLEM

Many uncertainties and interrelated economic and political factors affect stock market both locally and globally. The secret of successful stock market forecasting lies in acquiring better results with minimum input data sets. Determining the set of significant factors for predictions with full accuracy is a big complicated task and hence continuous and regular stock market analysis is very essential. More specifically, the stock markets' movements are analysed and predicted in order to retrieve knowledge that could guide the investors in terms of buying and selling timings. It also helps the investor to make money through his investment in the stock market. More importantly the reasons for analysing stock market movements predictions is to acquire more knowledge that could help the investor in terms of buying and selling timing and to make money through investments in the stock market. The current study is important milestone in predicting stock market from the perspective of mathematical modeling.

GOALS AND OBJECTIVES

Goal:

The main focus of the study is to examine the usefulness of the existing Stochastic models to classify the financial data by including **a set of new operators and combinatorial structure of statistical distributions** into the system of **parametric** and non – parametric models and to develop other associated intelligent techniques to enhance their capabilities in effective forecasting of stock market by considering the risk – return series and Volatility coefficients obtained from the data.

The Objectives of the current study are:

1. To utilize scale mixture distribution for extracting the features of economic factor. ***(model:1)***
2. To design a new Lehmann type - 1 Exponential Mixture distribution by combining Gaussian and generalized exponential distributions for estimation of financial volatility.(***model 2)***

3. Ranking volatility of various financial investments using R – procedure. (***model 3)***
4. To build Gaussian and Laplacian HMM to train the financial data for predicting optimum risk-return sequence.(***model 4)***
5. To develop a combinatorial data processing tool by integrating a suitable 'Data Reduction Technique' with an efficient classifier.(***model 5)***
6. To devise discrete stock-well transform approach for setting up extrapolation techniques for Volatility prediction.(***model 6)***

METHODOLOGY

Choice of a few known Computational models

GARCH family of techniques are found to capture mainly time varying volatility, volatility clustering, mean revision and asymmetric volatility with a reasonable level of accuracy. However, to accommodate both local and global variations and to estimate historical volatility, implied volatility, risk – return series etc, it is advantageous to take up studies of HMM, structural risk minimization based techniques such as SVM, SVM regression classifiers with reduced dimensions of data sizes using PCA like techniques for dimensionality reduction. Hence, the following objectives have been taken up as the main subject of study in this work.

Acquisition and Pre-Processing Stock Market Data

The raw data obtained from the source shows much inconsistency in terms of sudden variation or sometimes very low variation over time. To preserve consistency, the data have been pre-processed (log data derivation or first difference etc) to make it well-conditioned so that the available models can be used for validation and comparative study purposes. The sources from where data has been derived and the model with which they have been tested for volatility have been described below:

To carry out model based studies of various stock markets, data have been derived from various agencies from the following perspectives:

1. Benchmark data such as S&P 500, BSE Index and DJIA etc. so as to check the efficacy of presently available models such as HMM, SVM – PCA etc, with some newly introduced metrics.
2. A few Nifty data sets (pertaining to Indian stock market) and S&P 500 data to check the efficiency of both available and the new mixture distribution based models.
3. A sample of TCS and CSIAC software real time data sets have been taken up for study for analysing the efficiency of mixed distribution fitting to obtain volatility estimates through their parameters.

EXPLANATION OF MATHEMATICAL MODELS IN PREDICTING VOLATILITY

The study objectives have been validated by the following mathematical approaches and the conceptual model has been given in the following figure 1

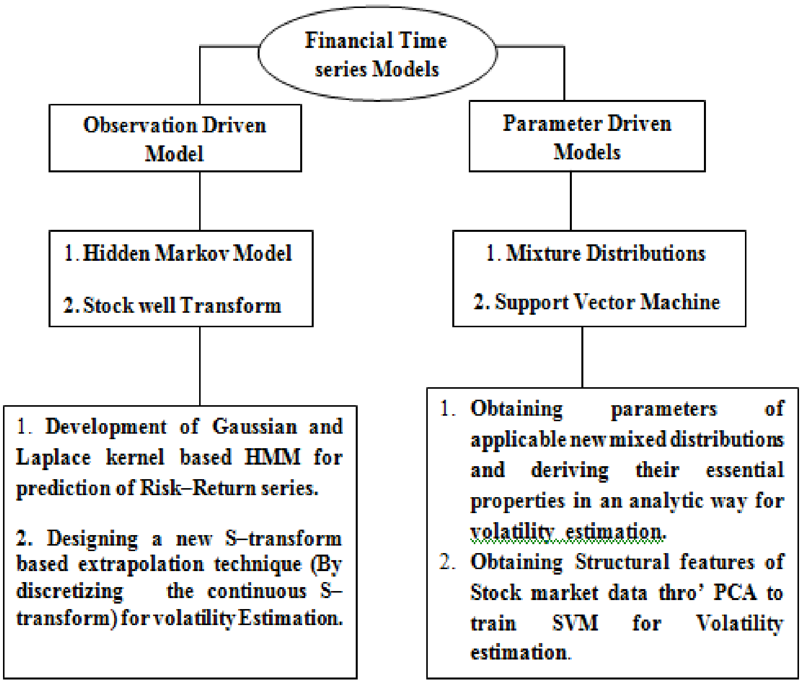

Figure:1 conceptual model

Volatility as a function of economic factor *(model:1)*

Movements in the stock market can have a profound economic impact on the economy and people. Volatility plays an important role in risk measurement and management. Though volatility is related to risk, it is not exactly the same. Risk shows undesirable outcome, but volatility measures positive outcome. Volatility is measured by using the variance between returns or by standard deviation from that same security or market index. Normal distribution of returns is symmetrical and it estimate the potential gains or losses related to each amount. Thus, historical volatility is used as a risk indicator. Returns are usually calculated from market close-close price changes.

Mixtures of normal distributions are well recognized in empirical finance. There exists a long history of modeling asset returns with a mixture of normal. Mixture of Normal distributions is proposed to accommodate the non-normality and asymmetry characteristics of financial time series data as found in the distribution of monthly rates of return.

Stock return volatility is fundamental to finance. Volatility often plays a crucial role in measuring the total risk of financial assets, evaluating option prices and conducting hedging strategies.[2-5] . Owing to the seminal works of Engle[6] and Bollerslev[7] about heteroskedastic return series models, it has become widely recognized in the academic finance literature that ARCH family are effective methods for volatility forecasting [8,9]. Poon and Granger [10] indicate in forecasting volatility in financial markets that times series volatility forecasting models can be explained by standard deviations, the SV model and ARCH and GARCH models. Empirical findings from studies conclude that GARCH is a more parsimonious model than ARCH, and GARCH (1, 1) is the most popular model for examining financial time series.

In this study, the volatility is predicted as a function of the economic factor, the mean economic ratio for a particular time period. There are many volatility models and forecasting methods. Some of these are Historical volatility models, implied volatility models, Autoregressive Conditional Heteroskedasticity models and models based on Artificial Neural Network. All these models are direct models. In these models, the influence of economic factors like price level uncertainty, the riskless rate of interest, the equity risk premium and the ratio of expected profit to expected revenue for the economy are not taken into account The main economic factor considered is the ratio of expected revenue to expected profit.

The scale mixture of Gaussian distribution [11] is used for modeling the stock return data. The volatility σ, a parameter of the normal distribution, is considered as a random variable z, which is the ratio of expected revenue to expected profit. Economic ratio z is assumed to follow the exponential distribution. The resultant distribution is fitted to Dow Jones Industrial Average (DJIA) data and Nifty50 data by estimating the parameters. It is observed that the scale mixture distribution is a better fit than that of GARCH for financial return series, whenever the volatility parameter is assumed to be a function of one of the economic factors, namely the ratio of expected profit to expected revenue. It is observed that there is a decrease in volatility, when there is an increase in the mean ratio of expected revenue to expected profit.[12]

Lehmann Type I Exponential Mixture Distribution for Volatility Estimation (*model 2)*

A new three-parameter family of mixture distribution is obtained by mixing Gaussian and Lehmann Type I Exponential distribution on (0,∞). [13,14] .

The probability density function (pdf) of the newly type of Lehmann Type I Exponential Mixture Distribution (LEMD) has been derived to be

$$P_X(x) = \frac{\alpha}{2}\sum_{n=0}^{\infty}\sqrt{\frac{2}{\lambda(n+1)}}(-1)^n\binom{\alpha-1}{n}\exp\left(-\sqrt{\frac{2(n+1)}{\lambda}}|x-\mu|\right) \quad \alpha,\lambda>0\,;-\infty<\mu<\infty$$

where the shape parameter $\alpha > 0$ is an integer. $\lambda > 0$ is the scale parameter and $-\infty < \mu < \infty$ is the location parameter. The shape parameter α controls the skewness and kurtosis of the distribution.

Properties of the distribution are studied. Survival function and hazard function are derived and the standard form of the distribution is obtained in this study. This distribution is fitted to the return data for 200 days (02/01/'12 to 16/10/'12) of the TCS Index(obtained from www.yahoofinance.com). In this data, σ, the volatility, follows Lehmann Type I Exponential distribution. The parameters are estimated by the method of maximum likelihood estimation . It is proved that LEMD has the maximum log-likelihood value and minimum AIC value than scale mixture of Gaussians which proves that LEMD is a better fit for this financial data. 1/λ gives the mean economic ratio.[15] .

A Ranking And Selection Approach For Volatility in Financial Market(*model 3)*

Gupta and Sobel [16], gave a multiple decision approach to the problem of selecting a subset from k given normal populations which include the best population. The population variances are unknown and the population means may be known or unknown. Based on a common number of observations from each population, a procedure R is defined which selects and assigns rank for the unknown population variances and finally gives the best population with the smallest variance. In this study, ranking and selection approach of Shanthi S. Gupta [17] is used to select a subset of stock market populations containing the population with the smallest variance, i.e., volatility.

This study concludes that the procedure R is used to rank the volatility for the DJIA , S&P 500, Oil, Spectrum and Mobile indices. Each population has sub populations. The sub populations are ranked according to their volatility and the best sub population from each population is selected. Considering the resultant sub populations as main populations, using R procedure, the best population in terms of minimum volatility is obtained. Also, the result coincides with Scale Mixture distribution and Lehmann Type I Exponential Mixture Distribution. On the basis of selection, a suitable proportion of investment can be decided by an investor according to his / her expectation. [18]

Gaussian and Laplacian HMM for Financial data analysis(*model 4)*

In recent years, a variety of forecasting methods has been formulated and put into practice for tock market analysis. Hidden Markov Model (HMM), [19,20] is extensively applied to predict stock market data. The stock market prediction problem is analogous to its inherent relation with time. HMM are based on a set of unobserved underlying states in the midst of which transitions can occur and each state is connected with a set of possible observations. The stock market can also be seen in an analogous manner. The fundamental states which establish the behavior of stock values are usually unseen to the investor. The transitions between these underlying states are based on company policy decisions, economic conditions etc. The visible effect which reflects these is the value of the stock. Clearly, the HMM confirms well to this real life scenario. The observation emission densities of the HMM, hidden states are typically modeled by means of elliptically contoured distributions, usually multivariate Gaussian or student's t densities.

In this study, Bivariate Gaussian density and Bivariate Laplace densities are used as observation emission densities to obtain risk-return state series.

The States 1 and 2 of the HMM are return and risk calculated from the data considered. The feature vectors are then clustered using $k-means$ clustering [21,22] and minimum distance algorithm used by He and Kundu [23] , to obtain the state sequence, to one of the N=1, 2 states. All 80 vectors are clustered in an analogous way and the resultant feature vectors $S_j = \{s_{jt}, t = 1, 2, 3, 4, 5, 6\}$ for $j = 1, 2,...., 8$ are assigned to one of the 2 states. Given a model $\lambda = (A,\Pi,B)$ and an observation sequence $O =\{o_1, o_2, ..., o_T\}$, the Viterbi algorithm finds the state-optimized likelihood function and the optimal state sequence. At each iteration, the optimal state sequences are assigned to the observation vectors of each training sample. The new states are again used to estimate new model parameters. The iteration proceeds until none of the state assignments change at the end of the maximization.

The daily closing values of S&P 500 and Nifty 50 are downloaded from www.yahoofinance.com for the period from 02,Jan 2009 to 31,Dec 2012 is taken for this study.

It is observed that for the S&P 500 and Nifty 50 data from Jan2013—Dec 2014, the predicted risk-return series using emission matrix of the optimum state sequence and the parameters of the Laplace HMM , return is more pronounced than risk.

This study concludes that the observation density is assumed to be multivariate normal and multivariate Laplace, two HMMs for the financial data are constructed and is observed that the Laplacian HMM is a better choice than the normal HMM. It is also more practical to assume Laplace in the place of normal because in practical situations the data cannot be symmetrical always. Thus, Laplacian HMM has a prominent role in financial data analysis.

Principal Component Analysis based Support Vector Machine Technique for Forecasting Stock Market Movements(*model 5)*

Conditional Volatility of stock market returns is one of the major problems in time series analysis. Support Vector Machine (SVM)[24] has been applied for volatility estimation of stock market data with limited success, the limitation being in accuracy in volatility feature predictions due to general kernel functions. However, Principal Component Analysis (PCA) [25] technique yields good characteristics that describes the volatility time series in terms of time varying risk estimates.

PCA, used as a tool in exploratory data analysis, which describes most of the variance of data and which was discovered by Karl Pearson in 1901, helps us to construct predictive models. After the development of computers, PCA has been widely used for pattern recognition problems due to its inherent capability of dimensionality reduction without losing information or capturing all features to generate a feature space.

In this study, especially for S&P 500 and Nifty50, BSE Index data which includes the data corresponding to this period (2008-2010) , three kernel functions namely linear, polynomial or radial basis function, Gaussian have been taken up for experimental study and prediction performance of the algorithm has been evaluated in terms of Normalized Mean Square Error (NMSE) and Normalized Mean Absolute Error (NMAE). It is observed that Gaussian kernel function is best suited for this hybrid, that minimises both NMSE and NMAE.

Also, PCASVM combined computational model is designed to forecast the deviations of stock market indices in terms of risk and return components. For classification of data by PCASVM, we have used 1(return) and -1(risk) as targets. In the proposed model, principal component features were extracted by processing the covariance matrices and SVM have been used to process the PCA feature vectors for support vector generations. Volatility of this data has been classified in to risk and return groups. The superiority of this computational model driven by Gaussian kernel function has been explained with the optimized parameter C and the quality metrics NMSE & NMAE. In this study, SVM and PCA SVM have been taken up for volatility comparison in terms of the above quality metrics and Mean & Variance. As a result, PCASVM is found to be a better model than SVM.[26]

Stock well -Transform based Stock Market data analysis for Volatility estimation (*model 6*)

In order to address the nonlinearity in stock market data, Fast Fourier Transform (FFT) is found to be a useful tool that delivers real time pricing, while allowing for a realistic structure of asset returns that include extra kurtosis and stochastic volatility[27] . Fourier transform of the entire time series contains information about the spectral components in a time series for a large class of practical applications. However, this information is inadequate. In these circumstances, FFT does not provide an exhaustive account in finance and a complete time frequency resolution of financial data leading to optimal solution which can forecast the future stock is scarcely seen in literature.

A spectral component of such a time series is clearly time dependent. It is always desirable to have a joint time frequency representation (TFR). Short Time Fourier Transform (STFT) exhibits frequency based features in short durations which may be found suitable for local time-frequency analysis of Fourier spectra. Similarly, discrete wavelet transform is also a useful tool for analyzing local features of any data in both time and frequency windows. However S-Transform due to its combined individual advantages of STFT and Continuous Wavelet Transform (CWT) provides a TFR along with frequency time dependent resolution, while maintaining the direct relationship with time averaging with Fourier spectra. One such application that requires this type of relationship is forecast of stock market indices. Hence, in order to have both time and frequency representation of the stock market data which are non-stationary in nature, S-transform as proposed by Stockwell etal, [28] 1996 is applied to different economic indicators to study various cycles involved in the business. The S-transform is a technique that localize time frequency spectral component, which combines the individual advantages of STFT and CWT with a Gaussian window with width scales inversely and height varies linearly with the frequency. Moreover, S-transform can be considered as the phase correction of CWT.

In this study, we have applied discrete S-transform for time series analysis of financial market, in order to obtain frequency based density estimates and improved volatility estimates. Gaussian kernel is capable of sensing the minor variations in the data. Better prediction accuracies of this transform have been ensured by validating the proposed approach through four financial data sets namely S&P 500, DJIA, Nifty 50 and BSE. However, in our observations, we find that there are sideway movements in long term trends which are usually called ‘whipsaws’. These are reverse signals in the long term trend. In this work, such types of noisy whipsaws have not been identified from real trend change signals as they are of low magnitude. However, since they are present in the outliers they can be identified and can be subject to post processing for further refinement.

CONCLUSION

Motivated by the advantages of various model based approaches in predicting a wide variety of stock market data, with accuracy the researcher suggested ***Hidden Markov Model*** as tracking approach in obtaining the transition probabilities from one state of return to others in terms of in-built quality metrics. The underlying ***Gaussian distribution*** paved the most appropriate way for estimating the chosen stock market data to form the kernel of the training scheme. To further conclude with authentically ***mixed distributions*** has been used for analysis to confirm suitability to form kernel functions for stock market data to account for historical nd implied volatilities based on research papers [29-32]

Intelligent computational techniques such as Support Vector Machine [33], Artificial Neural Network [34,35] and Genetic Algorithms are successfully applied to financial time series problems to obtain useful results with a reasonable level of accuracy. These research works have been referred to implement a classifier with suitable dimensionality reduction technique.

Volatility of stock market can very well predicted by the above mentioned select mathematical models which is a holistic and in depth understanding of usage of mathematical models in predicting stock market volatility / risk-return.

ACKNOWLEDGEMENT

Two of the authors wish to thank Department of Science & Technology, Government of India, New Delhi for financial sanction towards this work under FIST Programme (No.SR/FST/MSI-107/2015).

REFERENCES

[1] Box G. Jenkins G.M., Reinsel G. "Time Series Analysis: Forecasting & Control", Prentice Hall,– 1994, 614 pages, 3rd edition, ISBN:0130607746, 9780130607744.

[2] Day Theodore E. and Craig M. Lewis (1988), "The behavior of the volatility implicit in the prices of stock index options", Journal of financial economics, 22, 103-122.

[3] Harvey & Whaley, 1991, "S&P 100 Index option volatility", The Journal of Finance, 46(4), 1551-1561.

[4] Hull J.C. and A. White (1987), "The pricing of options on Assets with stochastic Volatilities", Journal of Finance, 42(2), 281-300.

[5] Poterba & Summers (1986), "The persistence of volatility and stock market fluctuations", American Economic Review, 76, 1142-1151.

[6] Engle, R.F., 1982, Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation, Econometrica 50, 987-1008.

[7] Bollerslev T. 1986, "Generalized Autoregressive Conditional Heteroskedasticity", Journal of Econometrics, 31:307-27.

[8] Philip Hans Franses and DickVan Dijk (1996), "Forecasting stock market volatility using (Non-linear) Garch Models", Journal of forecasting, vol.15, 229-235.

[9] Li. W.K. & Mak T.K. (1994), "On the squared residual auto correlations in non-linear time series with conditional heteroskedasticity", Journal of Time Series Analysis, 15, 627-636.

[10] Poon S. & Granger CWJ (2003), "Forecasting volatility in financial market: a review" Journal of Economic Literature, vol.41, pp:478–539.

[11] Andrews and Mallows, 1974, Scale mixtures of normal distributions, Journal of Royal Statistical Society Series B, vol.36, no.1, pp:99-102.

[12] R.Seethalakshmi, V.Saavithri, C.Vijayabanu, "Gaussian Scale Mixture Model for Estimating Volatility as a Function of Economic Factor", World Applied Sciences Journal, Vol.32, No.6, pp. 1035-1038, 2014.

[13] Gupta R.C, Gupta R.D & Gupta P.L, 1998, Modeling failure time data by Lehman Alternatives, Communications in Statistics-Theory and Methods, 27(4), 887–904.

[14] Gupta and Kundu, 1999, Generalized Exponential distributions, Australian and Newzeland Journal of statistics, vol.41, pp:173-188.

[15] [R.Seethalakshmi, V.Saavithri, "Parameter Estimation of Lehmann Type I Exponential Mixture Distribution", International Journal of Applied Engineering Research, ISSN 0973- 4562, Vol.10, No.4, pp. 8903-8912, 2014

[16] Shanthi S. Gupta and Milton Sobel (Dec.1962), "On the smallest of several correlated F Statistics", Biometrika , vol.49, no.3/4, 509-523.

[17] Shanthi S. Gupta and Milton Sobel (Dec.1962), "On selecting a subset containing the population with the smallest variance", Biometrika, vol.49, no.3/4, 495-507.

[18] R.Seethalakshmi, V.Saavithri, C.Vijayabanu, "A Ranking and Selection Approach for Volatility in Financial Market", Global Journal of Pure and Applied Mathematics, ISSN 0973-1768, Vol.11, No.5, pp. 3109-3119, 2015.

[19] L. Rabiner and B. Juang (1993), "Fundamentals of speech recognition", Prentice Hall, Englewood Cliffs, NJ.

[20] L.R. Rabiner (1993), "A tutorial on HMM and selected applications in speechRecognition", In:[WL], proceeding of IEEE, vol.77(2), pp:267-296.

[21] Teknomo Kardi "K- Means Clustering Tutorials", http: // people.revoledu.com / Kardi / tutorial / kmean / index.html.

[22] Juang B. and Rabiner L.R. (1990), The Segmental K –Means Algorithm for Estimating Parameters of Hidden Markov Models, IEEE Trans. Acoustic Speech Signal Processing vol.ASSP–38, no.9, 1639-1641.

[23] He Y. and Kundu A. (1991), 2-D Shape Classification Using Hidden Markov Model, IEEE Trans. Patt. Anal. Machine Intell. vol.PAMI-13, no.11, 1172-1184.

[24] Wei Huang, Yoshiteru Nakamori, Shou-Yang wang (2005), "Forecasting stock market movement direction with SVM", Computers & Operations Research 32, pp:2513 - 2522.

[25] Chitroub (2005), "Neural network model for standard PCA and its variants applied to remote sensing", International Journal of Remote Sensing, 26, 2197–2218.

[26] R.Seethalakshmi, V.Saavithri, V. Badrinath, C.Vijayabanu, "PCA based Support Vector Machine technique for Volatility forecasting", IJRET: International Journal of Research in Engineering and Technology, eISSN: 2319-1163 / pISSN 2321 - 7308.

[27] Cerny A. (2004), "Introduction to Fast Fourier Transform in Finance", Tanaka Business school Discussion Papers, London.

[28] R.G. Stockwell, L. Munisha and R.P. Lowe (April 1996), "Localization of the complex spectrum; The S-transform", IEEE Trans, Signal Processing, vol.44, no.4, pp:998-1001.

[29] Shuo Han, Rung-Ching Chen (2007), "Using SVM with Financial Statement Analysis for Prediction of Stocks", Communications of the IIMA, vol.7, Issue 4, pp:63-72.

[30] Guresen, Erkam, et al. (2011), Using artificial neural network models in stock market index prediction, Expert Systems with Applications 38(8),10389-10397.

[31] Kamruzzaman, J. and R.A. Sarker (2004), ANN-based forecasting of foreign currency exchange rates, Neural Information Processing - Letters and Reviews 3(2).

[32] Asai M. (2009), "Bayesian Analysis of Stochastic Volatility Models with Mixture-of- Normal Distributions", Mathematics and Computers in Simulation, vol.79, no.8, pp:2579-2596.

[33] Zhang M.H. and Cheng Q.S. (2005), An Approach to VaR for Capital Markets with Gaussian Mixture, Applied Mathematics and Computation, 168, 1079-1085.

[34] Barndorff-Nielsen O.E. (1997), Normal inverse Gaussian distributions and stochastic volatility models, Scandinavian Journal of Statistics^ 24, 1-13.

[35] William J. Reed (2006), "The Normal-Laplace distribution and its relatives", In Advances in Distribution Theory, Order Statistics and Inference pp:61–74. Birkhäuser, Boston.

[36] http://www.yahoofinance.com.